Nombre: …… Paralelo: …… Profesor: ……

Banco de Preguntas: Función Racional

1. Let $f(x) = \frac{1}{x}$, $x \neq 0$.

(a) Sketch the graph of f.

(2)

The graph of f is transformed to the graph of g by a translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

(b) Find an expression for $g(x)$.

(2)

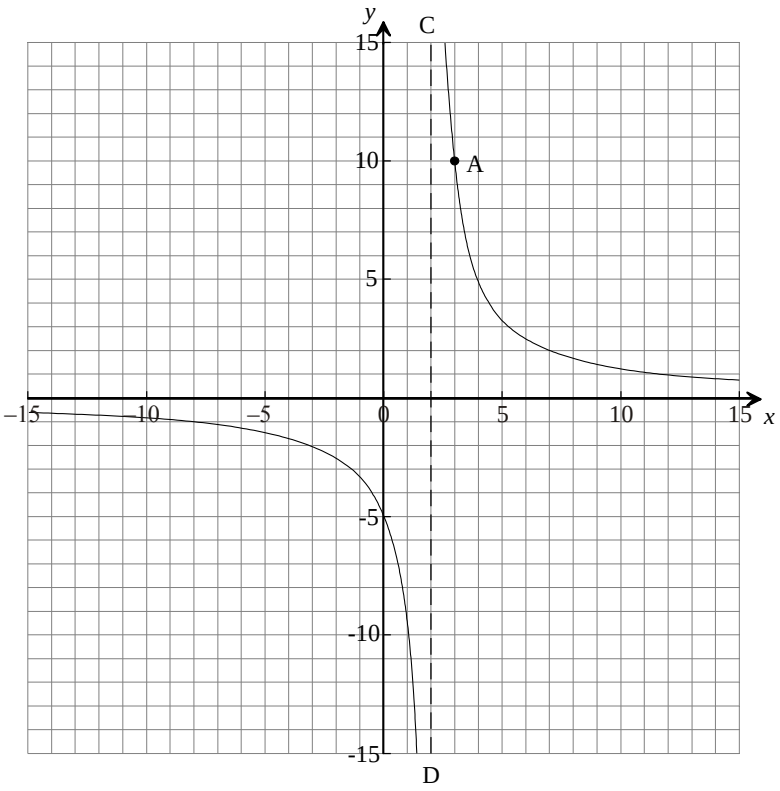
(c) (i) Find the intercepts of g.

(ii) Write down the equations of the asymptotes of g.

(iii) Sketch the graph of g.

(10)
(Total 14 marks)

2. (a) The diagram shows part of the graph of the function $f(x) = \frac{q}{x - p}$. The curve passes through the point A (3, 10). The line (CD) is an asymptote.

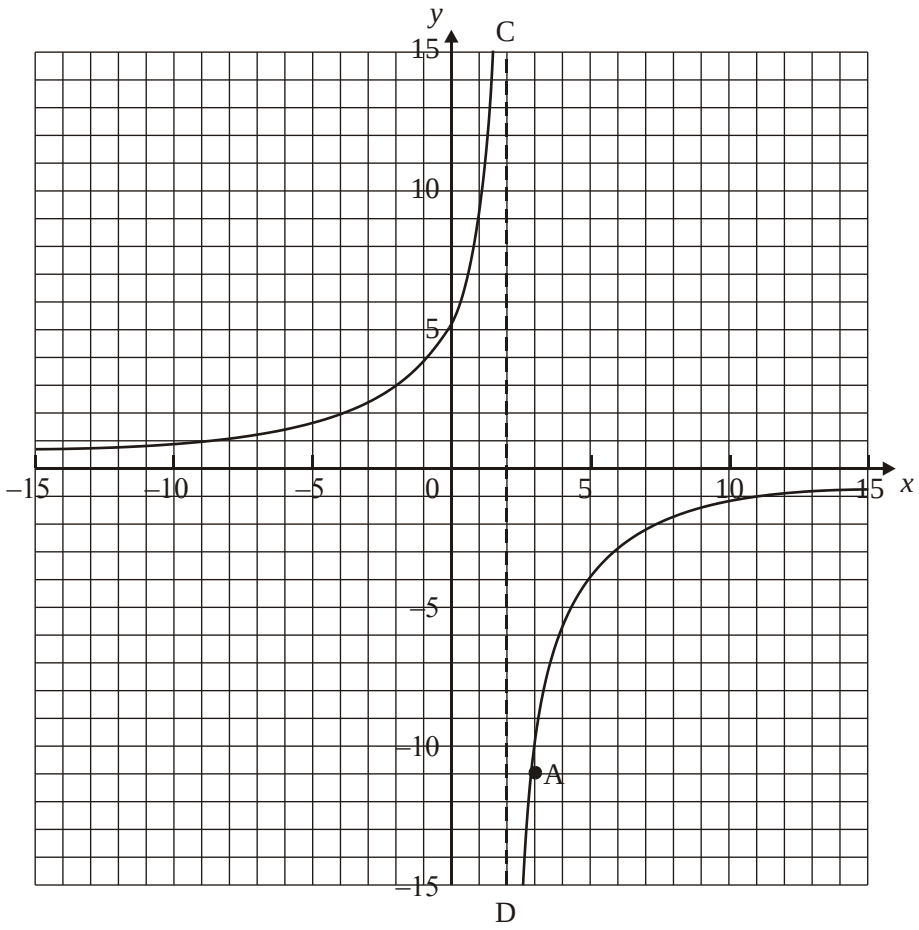


Find the value of

(i) p;

(ii) q.

(b) The graph of $f(x)$ is transformed as shown in the following diagram. The point A is transformed to A′ (3, –10).



Give a full geometric description of the transformation.

Working:

Answers:

(a) (i) .....

(ii) .....

(b) .....

.....

(Total 6 marks)

3. Consider the functions f and g where $f(x) = 3x - 5$ and $g(x) = x - 2$.

(a) Find the inverse function, f^{-1}.

(3)

(b) Given that $g^{-1}(x) = x + 2$, find $(g^{-1} \circ f)(x)$.

(2)

(c) Given also that $(f^{-1} \circ g)(x)$ $\frac{x+3}{3}$, solve $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$.

(2)

Let $h(x) = \frac{f(x)}{g(x)}$, $x \neq 2$.

(d) (i) **Sketch** the graph of h for $-3 \leq x \leq 7$ and $-2 \leq y \leq 8$, including any asymptotes.

(ii) Write down the **equations** of the asymptotes.

(5)

(Total 12 marks)

4. Let $g(x) = 3x - 2$, $h(x) = \frac{5x}{x-4}$, $x \neq 4$.

(a) Find an expression for $(h \circ g)(x)$. Simplify your answer.

(b) Solve the equation $(h \circ g)(x) = 0$.

(Total 6 marks)

5. Consider the functions $f(x) = 2x$ and $g(x) = \frac{1}{x-3}$, $x \neq 3$.

(a) Calculate $(f \circ g)(4)$.

(b) Find $g^{-1}(x)$.

(c) Write down the domain of g^{-1}.

(Total 6 marks)

6. The function f is given by

$$f(x) = \frac{2x+1}{x-3},\ x \in \mathbb{R},\ x \neq 3.$$

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$.

(2)

(ii) Find the vertical asymptote of the graph.

(1)

(iii) Write down the coordinates of the point P at which the asymptotes intersect.

(1)

(b) Find the points of intersection of the graph and the axes.

(4)

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines.

(4)

(Total 12 marks)

7. Consider the function $f(x) = \frac{16}{x-10} + 8,\ x \neq 10$.

(a) Write down the **equation** of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(2)

(b) Find the

(i) y-intercept;

(ii) x-intercept.

(2)

(c) Sketch the graph of f, clearly showing the above information.

(4)

(d) Let $g(x) = \frac{16}{x},\ x \neq 0$.

The graph of g is transformed into the graph of f using two transformations.

The first is a translation with vector $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$. Give a full geometric description of the second transformation.

(2)

(Total 10 marks)